Tri-Vertices & SU(2)'s

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Introduction

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- N=2 gauge theories in 3+1 dimensions

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- revisit - study a class made out of SU(2)'s

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- Vector Multiplet Moduli Space
- Hypermultiplet Moduli Space
- revisit - study a class made out of SU(2)'s
- interesting class of HyperKahler manifolds

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- H plet moduli space is classical
- argue it has many interesting features

From Quivers to Skeleton Diagrams

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- each node - V plet
- each line - H plet, bifundamental
- extend to other matter?

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- restrict to 3-valent vertices and lines representing SU(2) gauge group

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- Lines & 3-valent Vertices

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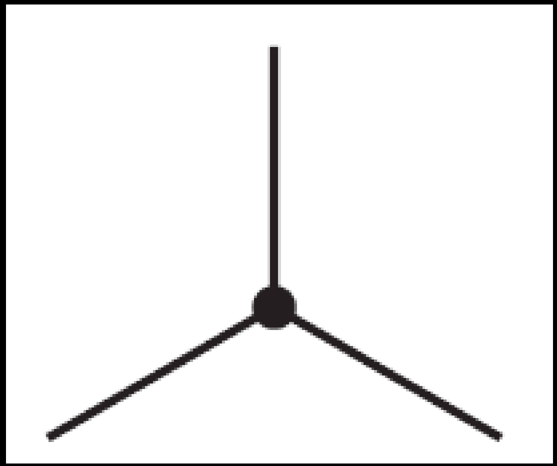
- Length of the line $L \sim 1/g^2$
- infinite line - global SU(2) symmetry

Example: 8 free 1/2 Hypers



$$W = \mathcal{Q}_{ijk}\mathcal{Q}_{i'j'k'}\left(m_1^{ii'}\epsilon^{jj'}\epsilon^{kk'} + \epsilon^{ii'}m_2^{jj'}\epsilon^{kk'} + \epsilon^{ii'}\epsilon^{jj'}m_3^{kk'}\right)$$

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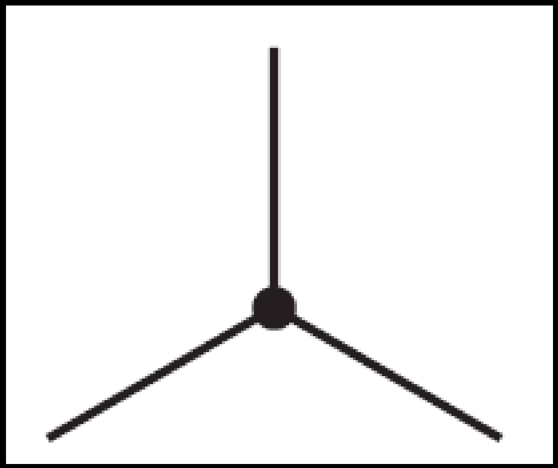
- global symmetry $SU(2)^3$



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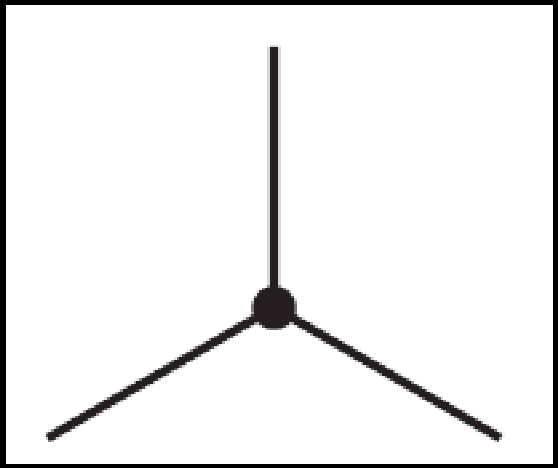
- global symmetry SU(2)^3
- No gauge group



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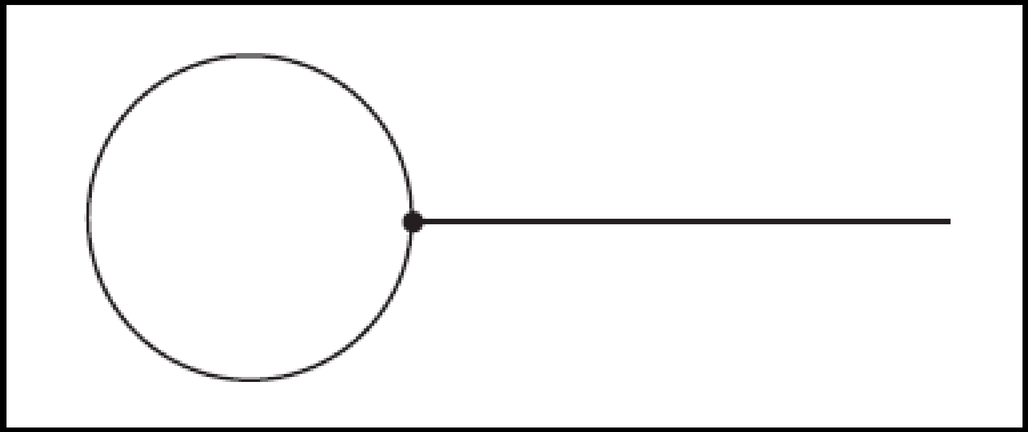

$$W = \mathcal{Q}_{abi}\mathcal{Q}_{a'b'j}\left(\phi^{aa'}\epsilon^{bb'}\epsilon^{ij} + \epsilon^{aa'}\phi^{bb'}\epsilon^{ij} + \epsilon^{aa'}\epsilon^{bb'}m^{ij}\right)$$

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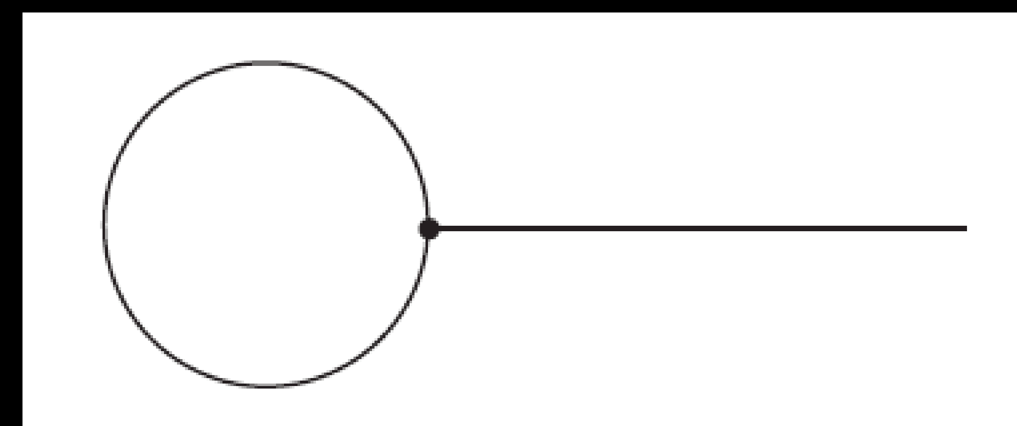

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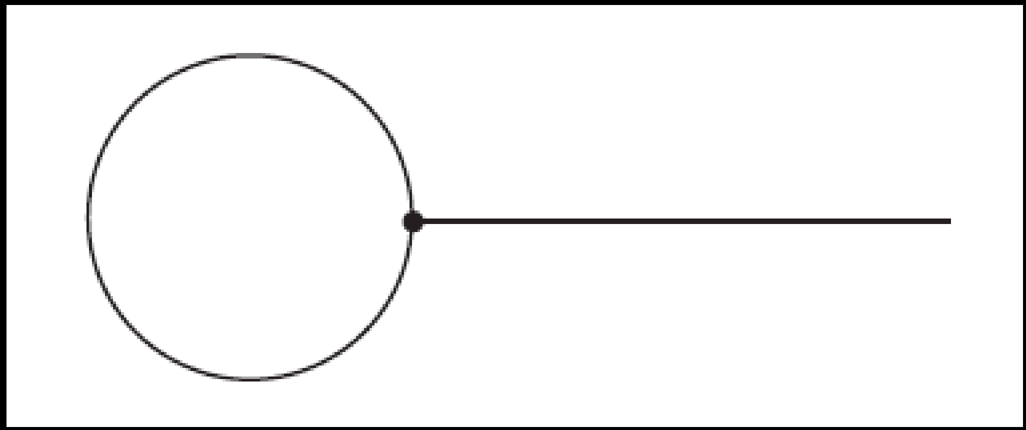

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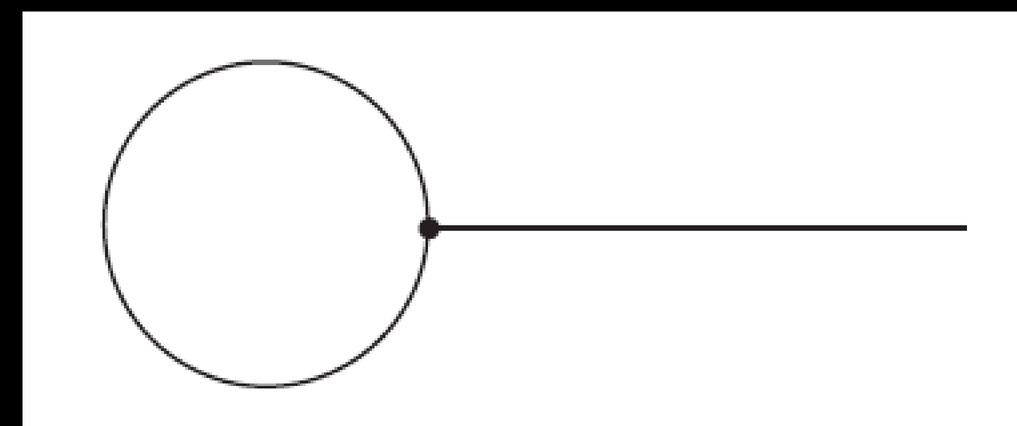

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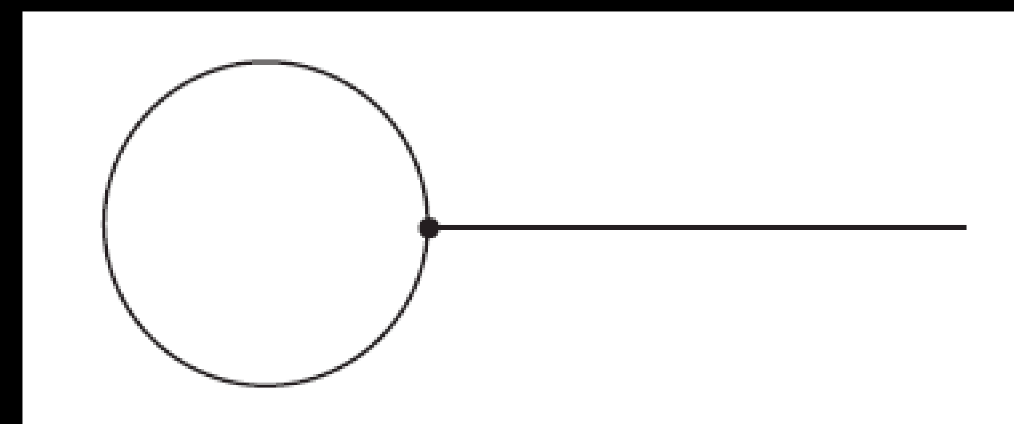

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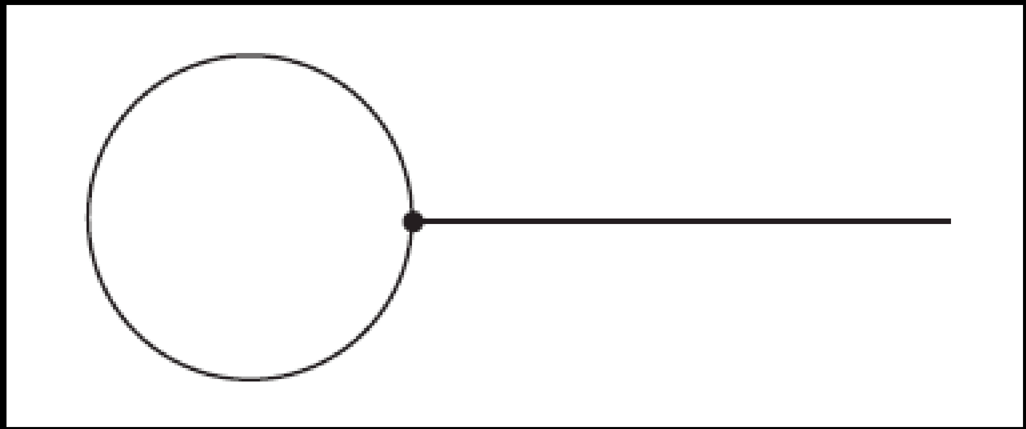
- possible N=2 breaking mass term



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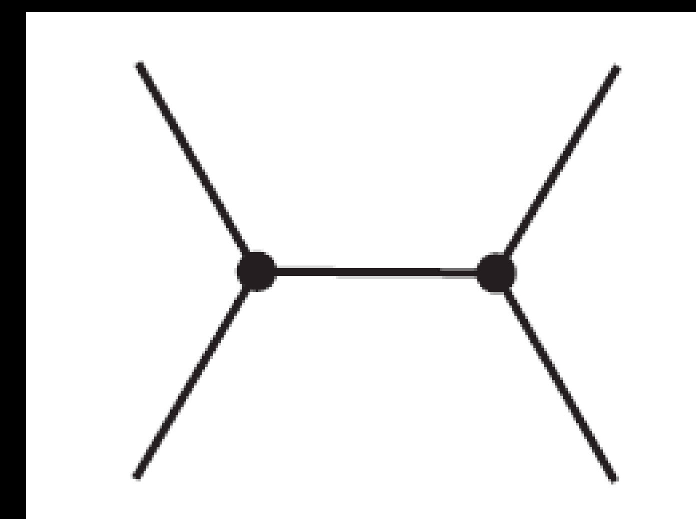
Example: SU(2) with 4 flavors



$$
\begin{aligned}
W = {} & \mathcal{Q}_{i_1 i_2 a}\mathcal{Q}_{i'_1 i'_2 a'}\left(m_1^{i_1 i'_1}\epsilon^{i_2 i'_2}\epsilon^{aa'} + \epsilon^{i_1 i'_1} m_2^{i_2 i'_2}\epsilon^{aa'} + \epsilon^{i_1 i'_1}\epsilon^{i_2 i'_2}\phi^{aa'}\right) \\
& + \widetilde{\mathcal{Q}}_{i_3 i_4 a}\widetilde{\mathcal{Q}}_{i'_3 i'_4 a'}\left(m_3^{i_3 i'_3}\epsilon^{i_4 i'_4}\epsilon^{aa'} + \epsilon^{i_3 i'_3} m_4^{i_4 i'_4}\epsilon^{aa'} + \epsilon^{i_3 i'_3}\epsilon^{i_4 i'_4}\phi^{aa'}\right) .
\end{aligned}
$$

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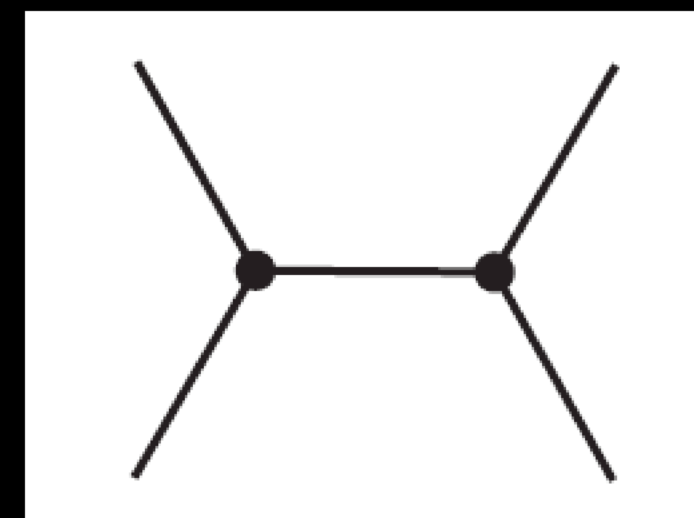

- 1 gauge group

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$$+\widetilde{\mathcal{Q}}_{i_3 i_4 a}\widetilde{\mathcal{Q}}_{i'_3 i'_4 a'}\left(m_3^{i_3 i'_3}\epsilon^{i_4 i'_4}\epsilon^{aa'} + \epsilon^{i_3 i'_3} m_4^{i_4 i'_4}\epsilon^{aa'} + \epsilon^{i_3 i'_3}\epsilon^{i_4 i'_4}\phi^{aa'}\right) .$$

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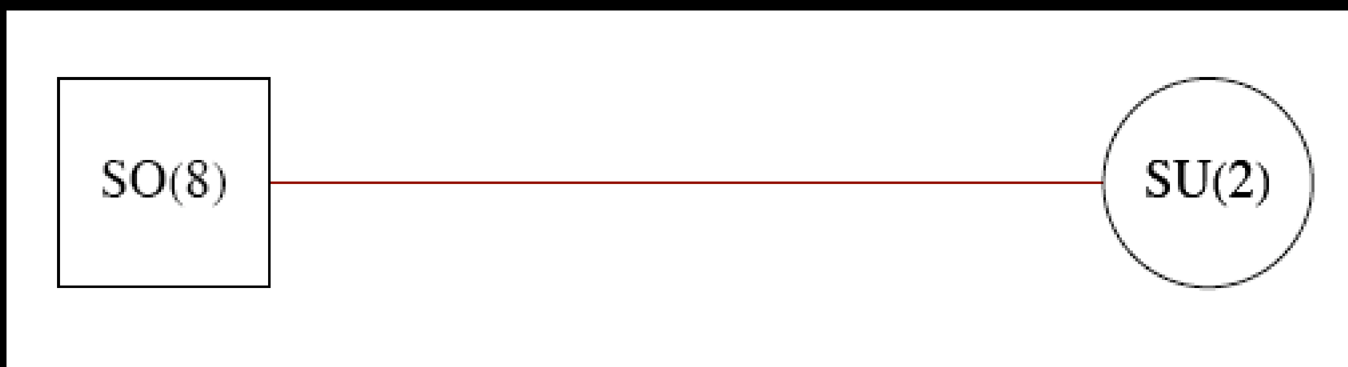

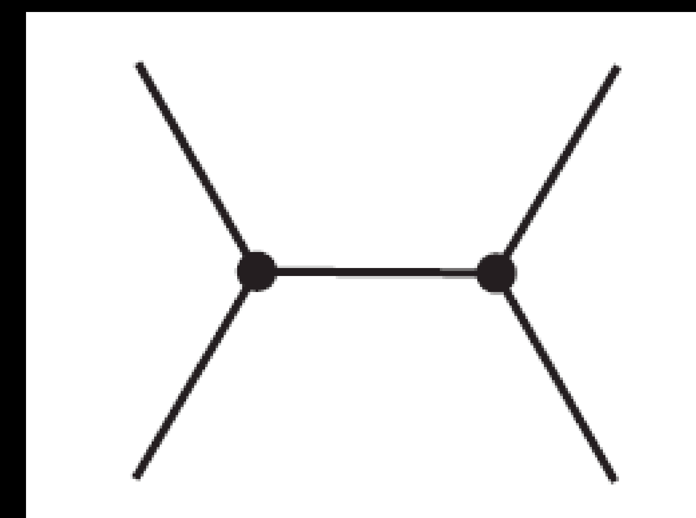

- 1 gauge group
- $SU(2)^4$ global symmetry

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- Feynmann diagram in phi cubed field theory but each diagram represents a unique Lagrangian

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- beta function is 0

An infinite class of N=2 SCFT’s

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- Kibble Branch

Look for generic results depending on g & e not on Lagrangian

Chiral Operators on Kibble Branch

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- Hilbert Series

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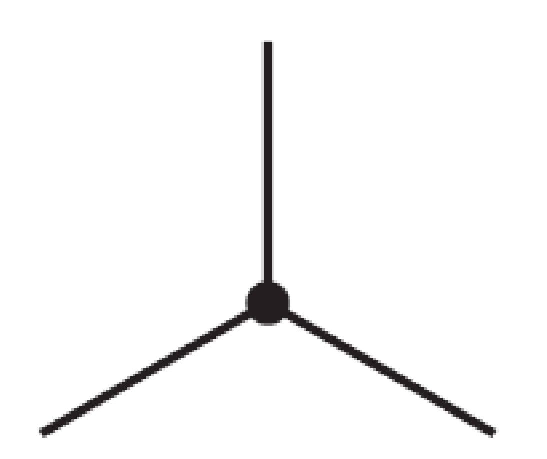
- HS(t, x_1, x_2, ..., x_e)

Example: g=0, e=3



$$g_{T_2}(t;x_1,x_2,x_3) = \mathrm{PE}\left[[1;1;1]t\right] = \prod_{\epsilon_i=\pm 1} \frac{1}{1-tx_1^{\epsilon_1}x_2^{\epsilon_2}x_3^{\epsilon_3}}$$

$$\begin{aligned} g_{T_2}(t;x_1,x_2,x_3) = \frac{1}{1-t^4}\sum_{n_1,n_2,n_3,m=0}^{\infty} &\left([2n_1+m;2n_2+m;2n_3+m]t^{2n_1+2n_2+2n_3+m}+\right.\\ &\left.[2n_1+m+1;2n_2+m+1;2n_3+m+1]t^{2n_1+2n_2+2n_3+m+3}\right) \quad .(4.3)\end{aligned}$$

Plethystics

$$PE\left[g(t_1,\ldots,t_m)\right] = \exp\left(\sum_{k=1}^{\infty} \frac{g(t_1^k,\ldots,t_m^k)}{k}\right)$$

$$g(0,\ldots,0) = 0,$$

Example: g=0, e=4

$$g_{N_c=2,N_f=4}(t, z_1, z_2, z_3, z_4) = \sum_{k=0}^{\infty} [0, k, 0, 0]_{SO(8)} t^{2k}$$

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- To compute the HS observe

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- Next decompose irreps of SO(8) to $SU(2)^4$

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- Next decompose irreps of SO(8) to SU(2)^4
- Use fugacity map

$$z_1 = x_1 x_2, \quad z_2 = x_2^2, \quad z_3 = x_3 x_2, \quad z_4 = x_4 x_2$$

HS: g=0, e=4

$$g_{N_c=2,N_f=4} = \frac{1}{1-t^4} \sum_{n_1,\ldots,n_4,m=0}^{\infty} \Big([2n_1+m; 2n_2+m; 2n_3+m; 2n_4+m] t^{2n_1+2n_2+2n_3+2n_4+2m} + [2n_1+m+1; 2n_2+m+1; 2n_3+m+1; 2n_4+m+1] t^{2n_1+2n_2+2n_3+2n_4+2m+4} \Big) \ . \quad (4.7)$$

Gluing 2 theories with (0,3) to form (0,4)

$$g(t, x_i, y_j) = \int \mathrm{d}\mu_G(z_k)\ g_1(t, x_i, z_k)\ g_{\mathrm{glue}}(t, z_k)\ g_2(t, y_j, z_k)$$

$$g_{\mathrm{glue}}(t, z_k) = \frac{1}{\mathrm{PE}\left[Adj(z_k)t^2\right]}$$

$$\int \mathrm{d}\mu_{SU(2)}(z)\ g_{T_2}(t; x_1, x_2, z)\ g_{\mathrm{glue}}(t, z)\ g_{T_2}(t; x_3, x_4, z)$$

permutation symmetry

permutation symmetry

- The result has S_4 symmetry - permutation of external legs (global symmetries)

Example: g=1, e=1

$$
\begin{aligned}
\mathcal{F}^{\flat}(t,z,x) &= (1 - t^2[2;0] + t^3[0;1])\mathrm{PE}\left[[2;1]t + [0;1]t\right] \\
&= \frac{1 - t^2(1 + z^2 + \frac{1}{z^2}) + t^3(x + \frac{1}{x})}{\left(1 - \frac{t}{x}\right)^2 (1 - tx)^2 \left(1 - \frac{t}{xz^2}\right)\left(1 - \frac{tx}{z^2}\right)\left(1 - \frac{tz^2}{x}\right)(1 - txz^2)}
\end{aligned}
$$

$$
\begin{aligned}
g_{\mathrm{tadpole}}(t,x) &= \frac{1 - t^4}{(1 - tx)(1 - \frac{t}{x})(1 - t^2)(1 - t^2x^2)(1 - \frac{t^2}{x^2})} \\
&= (1 - t^4)\mathrm{PE}\left[[1]t + [2]t^2\right] \\
&= \frac{1}{1 - t^4}\sum_{n_1,n_2,m=0}^{\infty}\left([2n_1 + m]t^{2n_1+m} + [2n_1 + m + 1]t^{2n_1+2n_2+m+3}\right)
\end{aligned}
$$

g=1, e=1 another method

$$
\begin{aligned}
g_{\mathbb{C}^2/\mathbb{Z}_2\times\mathbb{C}^2}(t,x) &= \frac{1}{2}\left[\frac{1}{\left(1-\frac{t}{x}\right)(1-tx)} + \frac{1}{\left(1+\frac{t}{x}\right)(1+tx)}\right] \times \frac{1}{\left(1-\frac{t}{x}\right)(1-tx)} \\
&= (1-t^4)\mathrm{PE}\left[[1]t+[2]t^2\right] \ . \qquad (5.8)
\end{aligned}
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g=1, e=1 another method

- two commuting adjoints

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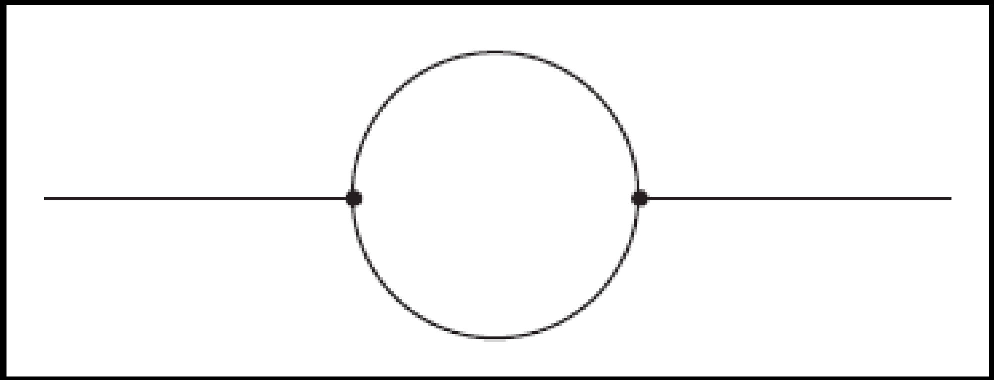
$$= (1-t^4)\mathrm{PE}\left[[1]t+[2]t^2\right] \ . \qquad (5.8)$$

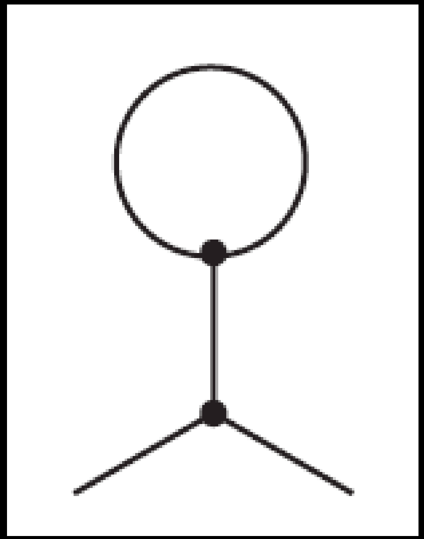
g=1, e=1 another method

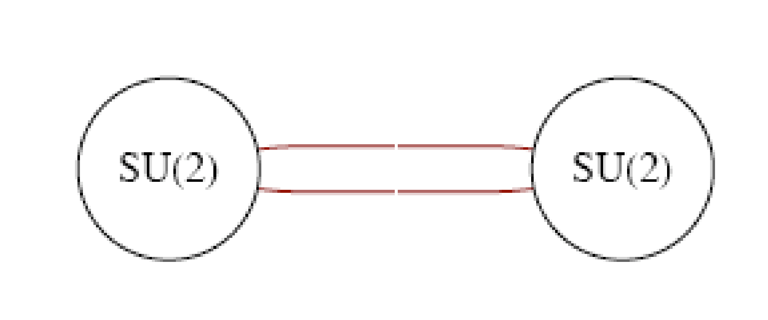
- two commuting adjoints
- symmetric product of 2 C^2 s

$$\begin{aligned} g_{\mathbb{C}^2/\mathbb{Z}_2\times\mathbb{C}^2}(t,x) &= \frac{1}{2}\left[\frac{1}{\left(1-\frac{t}{x}\right)(1-tx)}+\frac{1}{\left(1+\frac{t}{x}\right)(1+tx)}\right]\times\frac{1}{\left(1-\frac{t}{x}\right)(1-tx)} \\ &= (1-t^4)\mathrm{PE}\left[[1]t+[2]t^2\right] \ . \end{aligned} \tag{5.8}$$

Example: g=1, e=2

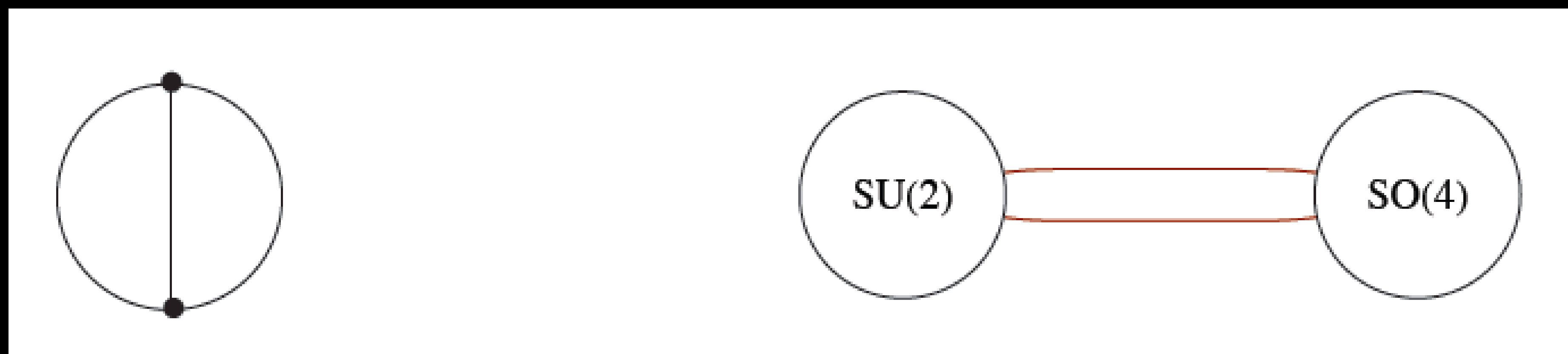






$$g_{A_1}(t, x_1, x_2) = \frac{1}{1-t^4} \sum_{n_1,n_2,m=0}^{\infty} [2n_1+m; 2n_2+m] t^{2n_1+2n_2+2m} + [2n_1+m+1; 2n_2+m+1] t^{2n_1+2n_2+2m+4} \ .$$

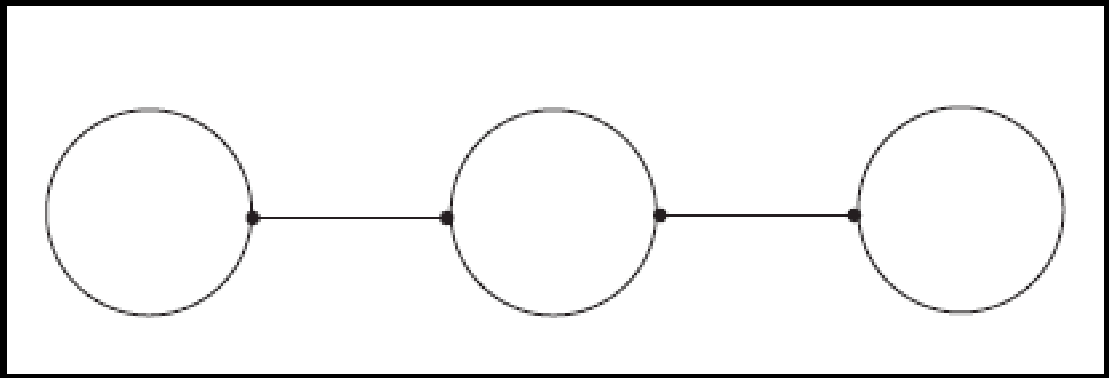
Example: g=2, e=0

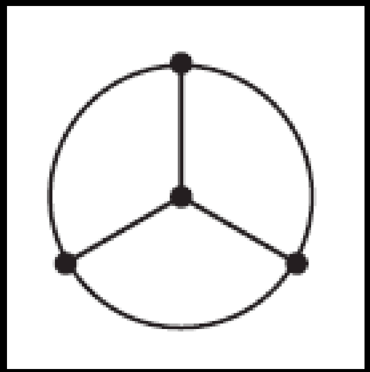


$\mathbb{C}^2/\hat{D}_3$

$$\begin{aligned}
g_D(t) &= \int \mathrm{d}\mu_{SU(2)}(z) g_{\mathrm{tadpole}}(t,z) g_{\mathrm{glue}}(t,z) g_{\mathrm{tadpole}}(t,z) \\
&= \oint_{|z|=1} \frac{\mathrm{d}z}{z}(1-z^2)\frac{(1-t^4)^2 \mathrm{PE}\left[2[1]_z t + 2[2]_z t^2\right]}{\mathrm{PE}\left[[2]_z t^2\right]} \\
&= \frac{1-t^8}{(1-t^2)(1-t^4)^2} \ .
\end{aligned}$$

examples: g=3, e=0





Duality statement

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- physics depends on g & e and not on the particular choice of the Lagrangian

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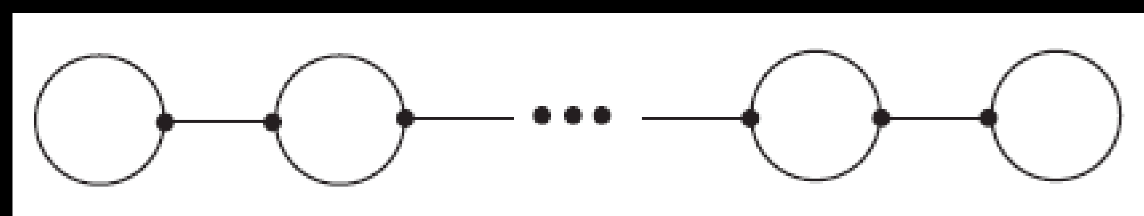
- Many to 1 correspondence

general case: (g,e)

$$g_{(g,e)}(t,x_1,\ldots,x_e) = \frac{1}{1-t^4}\sum_{n_1=0}^{\infty}\cdots\sum_{n_e=0}^{\infty}\sum_{m=0}^{\infty}\Big([2n_1+m,\ldots,2n_e+m]\,t^{2n_1+\ldots+2n_e+\chi m}$$
$$+[2n_1+m+1,\ldots,2n_e+m+1]\,t^{2n_1+\ldots+2n_e+\chi m+\chi+2}\Big)\ , \quad (7.1)$$

any g, e=0

$$g_{(g,e=0)}(t) = \frac{1-t^{4g}}{(1-t^4)(1-t^{2g-2})(1-t^{2g})} = \frac{1+t^{2g}}{(1-t^4)(1-t^{2g-2})}$$



$\mathbb{C}^2/\hat{D}_{g+1}$

any g, e=1

$$
\begin{aligned}
g_{(g,e=1)}(t,x) &= \frac{1}{1-t^4}\sum_{n,m=0}^{\infty}\left([2n+m]\,t^{2n+\chi m} + [2n+m+1]\,t^{2n+\chi m+\chi+2}\right) \\
&= (1-t^{2\chi+2})\mathrm{PE}\left[[2]t^2 + [1]t^{\chi}\right] \ ,
\end{aligned}
$$

- complete intersection

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- triplet of dimension 2; doublet of dimension 2g-1
- one relation of degree 4g

generators of the moduli space

$$\mathrm{PL}\left[g_{(g,e)}(t,x_1,\ldots,x_e)\right] = \left([2;0;\ldots;0]+[0;2;\ldots;0]+\ldots+[0;0;\ldots;2]\right)t^2 + [1;1;\ldots;1]t^{\chi}+\ldots \quad (7.11)$$

generators

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- 2^e at dimension 2g-2+e

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- with $SU(2)^e$ isometry

Summary

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- properties of H plet moduli space (Kibble branch)
- Special class of HyperKahler manifolds with $SU(2)^e$ isometries
- Multi-ality: Physics depends on (g,e)

Thank you!